The paper of Ferrers (*Q.J.M.*, vol. xii., 1873) appeared six years after the publication of Thomson and Tait's "Natural Philosophy," but it is curious that there is not in the second edition (1883) of that treatise any hint of the existence of the peculiar systems of which Ferrers gave examples. In his review of the second edition of the "Natural Philosophy" (NATURE, vol. xx., 1879), Clerk-Maxwell, in his own inimitable way, directed attention to the introduction in that work of the method of Lagrange. "The two northern wizards were the first who, without compunction or dread, uttered in their mother tongue the true and proper names of those dynamical concepts which the magicians of old were wont to invoke only by the aid of muttered symbols or inarticulate equations." The spell of the "northern wizards" was not free from defect, but either their good fortune or their instinct preserved them from the examples in which the use of an incantation insufficiently guarded in its terms might have led to disaster.

This has not been the fate of every wielder of the magic wand of Lagrange. In the first issue, about twenty years ago, of an important treatise on "Rational Mechanics" a discussion of the motion of a coin rolling and spinning on a horizontal table appeared, in which the method of Lagrange was used with erroneous results. The error led the distinguished author to the invention of a new method, in which a set of general dynamical equations, which could be used instead of those of Lagrange, and were applicable in all cases, was set up. The error thus had the fortunate effect of enriching dynamical science. It is not, alas! the fate of all who make mistakes to rise on the stepping-stones of their errors to higher and better things.

In our opinion, recourse is had to the method of Lagrange in far too many cases. The student flies to it on the appearance of the least difficulty. A proper training in dynamics, which should be experimental as well as mathematical, would give students the power of solving problems of all kinds by the direct application of first principles. The use of Lagrange's equations does not develop this—indeed, it has a directly contrary effect. In this power the dynamical students and graduates of our universities are sadly deficient.

A good account of Lagrange's method is followed in Prof. Lamb's book by an exposition of Hamilton's dynamical method, and this in its turn leads to Jacobi's discussion of the integration of Hamilton's canonical equations by means of the complete integral of Hamilton's partial differential equation fulfilled by his so-called principal function S. The corresponding function S' connected with S by the relation $S+S'=\Sigma(pq)$ does not seem to be mentioned. A partial differential equation similar to, and yet curiously different from, that for S also holds for it. S is a function of the qs, t, and as many constants, which depend on the initial co-ordinates of the system, as there are independent co-ordinates; S' is a function of the ps, t, and the same number of constants as before, which, however, depend on the initial motion.

One or two examples of the solution of these partial differential equations—for example, Jacobi's discussion of the elliptic motion of a planet referred to three co-ordinates—would have added to the interest of what is in itself a very interesting chapter of an excedingly interesting book. A. GRAY.

Maya Civilisation.

The Inscriptions at Copan. By Sylvanus Griswold Morley. (Publication No. 219.) Pp. xii+643+33 plates. (Washington: The Carnegie Institution of Washington, 1920.)

IN this large quarto volume of more than six hundred pages, efficiently illustrated, Dr. S. G. Morley has produced a work which may justly be regarded as of the highest importance. It is necessary to enforce this statement, almost with a feeling of shame, because there are so few individuals in this country who have the faintest idea what enormous strides have been made in the elucidation of Central American archæological problems since our own countryman, Dr. Maudslay, published the results of his explorations in a series of volumes, which Dr. Morley generously, but no less justly, describes as "easily the most important field contribution to Maya archæology." Those volumes might have been expected to give a lead to British archæologists and explorers; as a matter of fact, not only did no one appear in this country to carry on Maudslay's work, but the fine series of moulds of the principal Central American carvings, which he made at the cost of enormous labour and expense, lie buried in the cellars of the Victoria and Albert Museum, and not even an approximately representative series of casts taken from them is available to British students.

It follows that such a work as this is almost the despair of the reviewer. Most people have heard of the Aztec, but the earlier Maya civilisation is familiar to few even by name. Yet the Maya had evolved a remarkably fine art, an elaborate hieroglyphic script, and a very highly developed calendrical system long before the first manifestations of culture made their appearance in the Valley of Mexico. To render a full appreciation and critique of Dr. Morley's book intelligible to the general reader, it would be necessary to write an introduction to Central American archæology; to deal with it from the purely scientific point of view is impracticable within the limits of an ordinary review, so many and important are the problems which the author raises. It is possible, therefore, to give only the merest sketch of the subject-matter, and to add a few remarks on the author's method of handling his material.

The early Maya settlements are found scattered throughout the forested region of Chiapas, eastern Guatemala, western British Honduras, and northern Honduras. It is clear that they had already been abandoned and veiled in thick jungle before the arrival of the Spaniards. The group with which Dr. Morley deals is situated in Honduras, in the valley of the Copan River, and covers a site of 30–35 sq. km. The ruins consist of sculptured monolithic *stelae* and "altars," temples, pyramids, "plazas," and a great complex, known as the "main structure," which exhibits evident signs of growth by accretion through many years. The site, apart from its architectural and artistic importance, is of paramount interest from the fact that it includes by far the greatest number of dated inscriptions, about 40 per cent. of those recorded for the whole of the Maya area, indicating that the "city" was in continuous occupation for about 350 years. It is the inscriptions which constitute the principal theme of the book, and the author has dealt faithfully with every inscribed monument, not only discussing the content of every inscription, but also giving a bibliography of each.

With regard to these inscriptions a word of explanation is necessary. The only portions which can be read with certainty are those connected with the Maya system of reckoning time. As much as 50 per cent. of the entire *corpus* of these inscriptions deals with calculations relative to the calendar; the rest are probably religious, with, perhaps, a small proportion of what may be historical data. But, while dated monuments may thus be arranged in sequence according to Maya chronology, no indisputable method has yet been reached of correlating that chronology with European time. Dr. Morley, in a very scholarly appendix, makes the attempt. His views are of particular interest to the present reviewer, who, some years ago, was rash enough to put forward a scheme on similar lines, producing, however, different results. At the same time, Mr. Bowditch, one of the greatest pioneers in the interpretation of Maya glyphs, working on other evidence, came to the same conclusion. Dr. Morley's theory would make the Copan monuments date from about the close of the second century A.D. to the early part of the sixth century, about 250 years later than the dating of Mr. Bowditch and the reviewer. Dr. Morley's argument does not carry absolute conviction to the reviewer; but discussion is perhaps unnecessary. The author gives good *prima facie* evidence that a certain group of glyphs, which accompanies the so-called "initial dates," relates to eclipses of the sun and moon. With this lead these glyphs should soon be deciphered, and the question will then be capable of proof on astronomical grounds. In this connection it might be mentioned that Dr. Morley has apparently overlooked, in his otherwise excellent bibliography, two papers by Mr. Richard C. E. Long dealing with the correlation of Maya and European time, which were published in *Man* during 1918.

The origin of the Maya civilisation is also closely argued by Dr. Morley in a well-written appendix, but here he lays himself open to criticism. His theory that the Maya came from the region of the Panuco Valley in the north is well argued on general grounds, but he certainly lays too much stress on dates which have been found on two small objects—the Tuxtla statuette and the Leyden plate. Considering the ritual and mythological importance of certain calendrical dates in the Maya religious system, it is impossible to regard specimens of this class as belonging to the same category as monuments which are so obviously commemorative as *stelae,* altars, and the like. The Tuxtla statuette, which bears almost the earliest date known in Maya chronology, he accepts as belonging to the area where it was found—*i.e.* considerably to the north of the "classical" Maya area—in spite of the fact that the glyphs are carved in the same style as the Dresden manuscript, which is recognised as belonging to a later period than the Copan monuments. The Leyden plate, which is carved in the earlier style, and was found in the southern Maya region, he assumes, is a specimen which has "wandered"—a hypothesis which he dismisses in connection with the Tuxtla statuette. This savours of "having it both ways," and his theory would have gained strength had he admitted that portable objects such as these are really *hors concours* as regards local dating.

One mistake—and that really of relative insignificance—may be recorded. Dr. Morley states that the few original monuments brought from

Copan to England by Maudslay, such as Altar R and part of the frieze of Temple II., are in the Victoria and Albert Museum. As a matter of fact, all the original sculptures collected by Maudslay were transferred to the British Museum shortly after their temporary deposition at South Kensington.

In conclusion, Dr. Morley's work is scientific and scholarly. As a scientific man and a scholar he aimed at perfection; he has achieved a landmark. Can higher praise be given?

T. A. JOYCE.

The History of Determinants.

The Theory of Determinants in the Historical Order of Development. By Sir Thomas Muir. Vol. iii., *The Period* 1861 *to* 1880. Pp. xxvi + 503. (London: Macmillan and Co., Ltd., 1920.) Price 35*s*. net.

THE period covered by this volume is perhaps the most important in the history of the subject. During that time three important branches of pure mathematics attained vast dimensions—invariant-theory, analytical geometry, and the general theory of algebraic numbers. In each of these, familiarity with determinants and their manipulation is essential, so a great many students mastered the determinant calculus, and applied it to a variety of problems. Incidentally, the properties of determinants aroused interest for their own sake; numerous papers dealing with them were published, and, above all, several treatises on the subject made their appearance, in which a compact notation replaced all the old cumbersome symbols, and practically all the theorems of the determinant calculus proper were expounded in a simple and orderly way.

What we may call the derivative part of the theory consists mainly of classifying determinants of special types. Thus in the present volume we have separate chapters on axisymmetric determinants, symmetric determinants, alternants, recurrents, Wronskians, Jacobians, etc. (sixteen chapters or so). Broadly speaking, these types come from two sources—either as the outcome of a particular research, not primarily concerned with determinants (thus continuants arose from the theory of continued fractions); or else from intrinsic characters belonging to the array from which the determinant is formed, as in the case of symmetric determinants. Of course, any special type of determinant can be specified *per se*; we are thinking rather of the way in which the discussion of particular types actually originated. A remarkable example is Smith's arithmetical determinant (p. 116), of n rows and columns, the value of which is the product $\phi(1).\phi(2)\ldots\phi(n)$, where $\phi(m)$ means the "totient" of m—namely, the number of integers prime to m and not exceeding it.

In a book such as this, one feature is almost sure to present itself. We shall find some excellent work unaccountably neglected, and results of first-rate importance only becoming generally known and appreciated after re-discovery, when their original authors are dead. The cases in this volume which strike the attention are those of Trudi and Reiss. Reiss's work on compound determinants goes back as far as 1867; the analysis of it on pp. 181–90 (in modern notation) shows its importance, and is worth study, because the theory of compound determinants is perhaps the one part of general determinant-calculus not yet fully reduced to its complete and simplest form.

In many applications the rank and elementary divisors of a determinant (or matrix) are of primary importance. The elementary divisors of an array depend upon the arithmetical or algebraical character of the field to which the elements of the array belong. Consequently, the determination of them does not properly belong to determinant-theory; on the other hand, the rank of an array is immediately calculable, on the assumption that we can calculate the "value" of any minor determinant, or, at any rate, decide whether it is or is not zero. It must often have been difficult for the author to decide when a theorem in matrices should or should not be considered one relating to determinants. Rank is referred to several times; apparently theorems about elementary divisors have been omitted. In the case when the elements of an array are ordinary integers it is clear from Smith's paper on linear indeterminate equations and congruences (1861) that he was then perfectly familiar with the existence and properties of elementary divisors; to that extent he anticipated the theory of Weierstrass, Frobenius, and others.

Opinions may differ about Sir T. Muir's choice of a subject on which to bestow his labour; some readers may regret that he did not select a branch of mathematics of a less circumscribed and subsidiary kind than determinant-theory undoubtedly is. But all will agree in admiring the ability and impartiality with which this labour of love has been accomplished, and rejoice to know that the fourth and final volume is nearly complete in manuscript. Histories of other branches of mathematics are badly wanted, and this work is a model of what such histories ought to be.

G. B. M.